**LECTURE 1**

“There is no right answer, but there are a lot of wrong answers.”

– Levy, when explaining the art of identifying models.

*Describing Time Series*

A **time series** is a set of observations taken at different points in time. The time variable, *t*, can be discrete or continuous, and the observations, , can also have discrete or continuous domains. When is drawn from a probability distribution, we call the time series a **stochastic process**. In theory, we would define the process by a probability distribution. If the variables are interrelated we would define a joint probability distribution:

If the process depends upon all past values this can contain infinitely many variables. Typically, we would simplify the description using marginal distributions:

|  |  |
| --- | --- |
| 1st order marginal distributions |  |
| 2nd order marginal distributions |  |
| … |  |

Describing the stochastic process in this way is not really feasible, especially if the probability distribution is changing with time. To make things simpler, we typically require a stochastic process to be **stationary**. This means that the probability distribution for is the same as that for any other segment of the time series (such as , obtained by shifting the time indices). When the probability series is not changing, we have:

|  |  |
| --- | --- |
|  |  |
|  |  |

(or)

The first statement tells us that the probability distribution for any given value in the times series is constant. This means that:

1. The expected value, , is constant for all *t*
2. The variance is constant for all *t*

The second statement tells us that the joint probability distributions between values separated by a constant lag is constant. This means that:

1. The autocovariance matrix, (where ), is constant for all *t*

These three properties apply to any series that is strictly stationary (as defined earlier: any series for which the probability distributions are constant over time). In practice, we will never know the exact probability distributions and will never be able to prove that they are constant over time. As such, we settle for using the three properties above to identify a stationary time series. Any time series which appears to satisfy these three requirements is said to be **weakly stationary**. In practice, when someone says “stationary” they mean “weakly stationary,” the difference between weak stationarity and strict stationarity being purely academic.

The mean and variance for samples from the time series are thus constant and defined mathematically as:

|  |  |  |
| --- | --- | --- |
| Name | Theoretical Definition | Estimate |
| Mean |  |  |
| Variance |  |  |

We also define the **autocovariance** between lagged samples:

which is estimated as expected:

As with covariance, we can scale the autocovariance to get an **autocorrelation** between -1 and +1. This is given as:

|  |  |  |
| --- | --- | --- |
| Name | Theoretical Definition | Estimate |
| Autocorrelation |  |  |

and we note that . The autocorrelation function indicates the strength of a linear relationship between variables at a given lag. Time series analysis often involves analysis of the autocorrelations. These can be plotted in a **correlogram**. A correlogram is often useful for helping us to identify an appropriate model for a stationary time series.

It is important to note that these concepts only apply when the time series is stationary. If the series is not stationary we will have to transform it so that it is. Our analysis can then be applied, and we can transform back to the original values when we want to forecast. In doing so, it will be important to learn how to test to see if a time series is stationary. We will also be looking at ways to identify appropriate models for different stationary series.

*Box-Jenkins Methodology*

The Box-Jenkins methodology seeks to fit a time series to a model in 3 general steps:

1. Identification – Select possible models based on statistical properties (ACF & PACF)
2. Estimate model parameters
3. Perform diagnostic checking to see if the model is an appropriate fit

If the model passes our diagnostic checks we can proceed with forecasting. Otherwise, we have to start over at step 1 with a new candidate model. We also note that the methodology applies specifically to ARIMA models which model univariate time series.

*Discrete Linear Stochastic Processes*

ARIMA models describe a general set of processes called **discrete linear stochastic processes** (DLSP). These are processes which can be written as:

The are weights applied to the random shocks or white noise variables, , which are iid random variables drawn from any distribution where and . For these types of processes, it can be shown that when the coefficients are summable, that is:

the series will be stationary. This condition allows us to calculate:

since:

since:

All of these are easily obtainable by the coefficients of the process and will be constant values that do not change with time. This means that the series has a constant mean, variance, and autocovariances, making it weakly stationary.

*MA(1) Processes*

A moving average (MA) process is a truncated version of a discrete linear stochastic process which contains only the first *q* terms. It is commonly written as:

The minus signs are a common convention. We can transform this to DLSP notation with:

For an MA(1) process:

We can use the DLSP coefficients to compute:

The autocorrelation values indicate that we would expect an ACF plot to show a single spike at and zero correlations everywhere else. We can also use the estimated autocorrelation to find the implied value of . This is done by taking:

and solving the quadratic equation for . Two roots will be found, and we take the one that satisfies , for reasons to be discussed later.

*MA(2) Process*

An MA(2) process given by:

can again be written in DLSP form with:

This provides the following theoretical statistics:

|  |  |
| --- | --- |
|  |  |

We thus expect an ACF plot for this process to have two spikes at and and be zero elsewhere.

If we have estimates for and we then have two equations involving the two unknown parameters, and . We can solve for these values and will only accept solutions that satisfy:

|  |  |
| --- | --- |
|  |  |

*MA(q) Process*

In general, an MA(q) process will have the first q autocorrelations non-zero with all remaining correlations zero.

*NOTE: We also discussed standard errors for estimated autocorrelations, but these are better described in the book notes.*

**LECTURE 2**

TODO:

**LECTURE 3**

*Mixed Autoregressive Moving Average (ARMA) Models*

Given the ARMA model:

We have the following conditions for stationarity and invertibility:

1. Stationary: roots of lie outside the unit circle
2. Invertibility: roots of lie outside the unit circle

*ARMA(1,1) Model*

TODO

*ARMA(p,q) Model*

TODO

*Partial Autocorrelation Function*

TODO

*Inverse Autocorrelation Function*

Given an ARMA model:

(which we call the **primal model**), we can swap the polynomials to get:

(which we call the **dual model**). The inverse autocorrelation function (IACF) is the defined as the ACF of the dual model.

TODO: include sample IACFs (page 9)

**LECTURE 7**